2/4/20

Scale Parameters (Continued)

Consider $X \sim \text{InvGauss}(\mu, \theta)$ (See P. 9 of the tables)

Let $Y = c \cdot X$

Note: $F_X(x) = \Phi\left[\frac{x-\mu}{\mu}\cdot\left(\frac{\theta}{x}\right)^{1/2}\right] + \exp\left(\frac{2\theta}{\mu}\right)\cdot\Phi\left[-\frac{x-\mu}{\mu}\cdot\left(\frac{\theta}{x}\right)^{1/2}\right]$

$$F_Y(y) = \Pr(Y \le y) = \Pr\left(X \le \frac{y}{c}\right)$$

$$= F_X(y/c) = \Phi\left[\frac{y/c-\mu}{\mu}\cdot\left(\frac{\theta}{y/c}\right)^{1/2}\right] + \exp\left(\frac{2\theta}{\mu}\right)\cdot\Phi\left[-\frac{y/c-\mu}{\mu}\cdot\left(\frac{\theta}{y/c}\right)^{1/2}\right]$$

$$= \Phi\left[\frac{y-c\cdot\mu}{c\cdot\mu}\cdot\left(\frac{c\cdot\theta}{y}\right)^{1/2}\right] + \exp\left(\frac{2\cdot c\theta}{c\cdot\mu}\right)\cdot\Phi\left[-\frac{y-c\cdot\mu}{c\cdot\mu}\cdot\left(\frac{c\theta}{y}\right)^{1/2}\right]$$

$\Rightarrow Y \sim \text{InvGauss}(c\cdot\mu, c\cdot\theta)$

Neither μ nor θ is a scale parameter.

Mixtures of r.v.'s

Part 1: Discrete mixture of discrete r.v.'s

Example: Bag of dice

40% are 4-sided

60% are 6-sided

Let X = r.v.r. the value of a roll of 1 randomly selected die from the bag.

Let A = rvr the value of a roll of a 4-sided die

B = ———————————————— 6-sided die

Then X is a 40%/60% mixture of A/B

Fact: If the parameter for X is a linear function of the density of X, then it will equal the weighted average of the corresponding parameters for the r.v.'s forming the mixture.

Examples: probabilities, expectations, derivatives

Non-example: variance

(Continue w/ dice example) Parameters

1) $Pr(X=3) = .4 \cdot Pr(A=3) + .6 \cdot Pr(B=3)$

$= .4\left(\frac{1}{4}\right) + .6\left(\frac{1}{6}\right) = 0.2$

Formalize: Let's introduce an "indicator" r.v.

$$I = \begin{cases} 4 & \text{if a 4-sided die is selected} \\ 6 & \text{—— 6-sided ——} \end{cases}$$

I	Pr
4	.4
6	.6

Note: $Pr(X=3 \mid I)$ is a r.v.

I	$Pr(X=3 \mid I)$	Pr
4	$1/4$	.4
6	$1/6$	.6

From above

$$Pr(X=3) = \underline{Pr(A=3)} \cdot \underline{(.4)} + Pr(B=3) \cdot (.6)$$

$$= \underline{Pr(X=3 \mid I=4)} \cdot \underline{Pr(I=4)}$$

$$+ Pr(X=3 \mid I=6) \cdot Pr(I=6)$$

$$\therefore Pr(X=3) = E\left[Pr(X=3 \mid I)\right]$$

2) $E[X] = (2.5)(.4) + (3.5)(.6) = 3.1$

I	$E[X \mid I]$	Pr
4	2.5	.4
6	3.5	.6

$\therefore E[X] = E\left[E[X \mid I]\right]$ Law of Total Expectation (Double-Expectation)

3) $E[X \wedge 2] = ?$

$$E[X \wedge 2] = E\left[E[X \wedge 2 \mid I]\right]$$

I	$E[X \wedge 2 \mid I]$	Pr
4	$7/4$	.4
6		.6

$X \wedge 2 \mid I=4$	Pr
1	$1/4$
2	$3/4$

Exp. Value $= 7/4$